

# 第1章：1端子対回路

## 1.5 リアクタンス関数の合成

キーワード：Cauer展開(連分数展開)

学習目標：Foster展開，Cauer展開でリアクタンス関数を合成することができる。

# 1 1端子対回路

## 1.5 リアクタンス関数の合成

### Cauer展開(連分数展開)

$$Z(s) = H \frac{(s^2 + \omega_1^2)(s^2 + \omega_3^2) \cdots (s^2 + \omega_{2n+1}^2)}{s(s^2 + \omega_2^2) \cdots (s^2 + \omega_{2n}^2)}$$

#### 【解法1】

$$Z(s) = a_0s + \frac{1}{a_1s + \frac{1}{a_2s + \frac{1}{a_3s + \cdots}}}$$

#### 【解法2】

$$Z(s) = \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\frac{b_2}{s} + \frac{1}{\frac{b_3}{s}}}}$$

# 【例】第1章【16】(1)

リアクタンス関数(インピーダンス)をCauer形【解法1】で実現せよ。

$$Z_1(s) = \frac{(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)}$$

$$= \frac{s^4 + 10s^2 + 9}{s^3 + 4s}$$

$$s^3 + 4s \overline{) \begin{array}{l} s^4 + 10s^2 + 9 \\ s^4 + 4s^2 \end{array} } \quad \begin{array}{l} s \\ 6s^2 + 9 \end{array}$$

$$= s + \frac{6s^2 + 9}{s^3 + 4s} = s + \frac{1}{\frac{s^3 + 4s}{6s^2 + 9}}$$

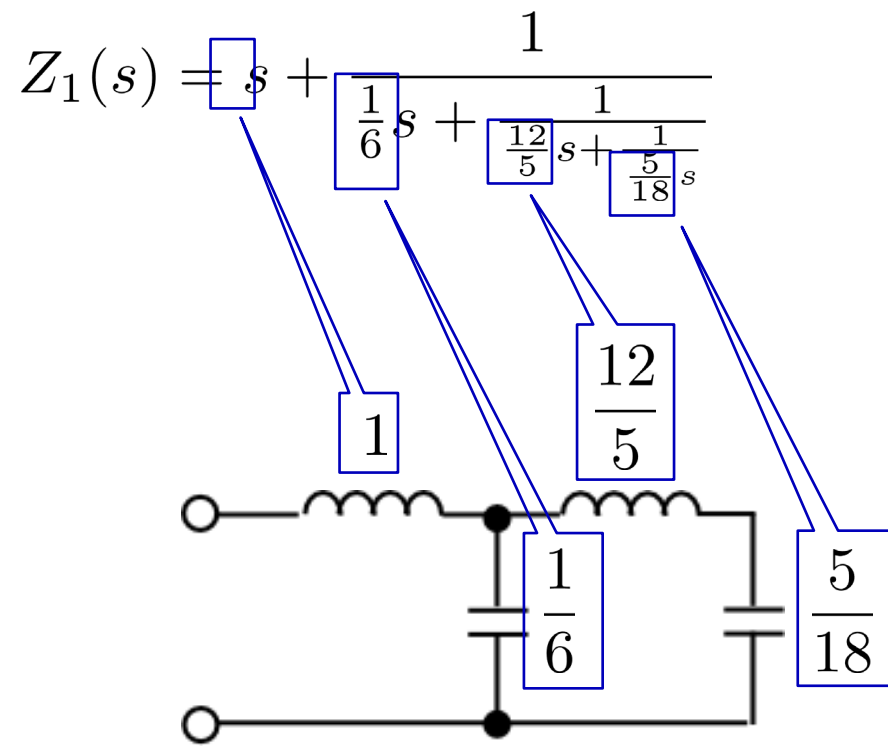
$$= s + \frac{1}{\frac{1}{\frac{1}{\frac{1}{6}s} + \frac{\frac{5}{2}s}{6s^2 + 9}}}}$$

$$= s + \frac{1}{\frac{1}{6}s + \frac{1}{\frac{5}{2}s}}}$$

$$= s + \frac{1}{\frac{1}{6}s + \frac{1}{\frac{12}{5}s + \frac{9}{\frac{5}{2}s}}}} = s + \frac{1}{\frac{1}{6}s + \frac{1}{\frac{12}{5}s + \frac{1}{\frac{5}{18}s}}}}$$

$$6s^2 + 9 \overline{) \begin{array}{l} \frac{1}{6}s^3 + 4s \\ \frac{1}{6}s^3 + \frac{3}{2}s \end{array} } \quad \begin{array}{l} \frac{1}{6}s \\ \frac{5}{2}s \end{array}$$

$$\frac{5}{2}s \overline{) \begin{array}{l} \frac{12}{5}s^2 + 9 \\ \frac{12}{5}s^2 \end{array} } \quad \begin{array}{l} \frac{12}{5}s \\ 9 \end{array}$$



# 【例】第1章【16】(1)

リアクタンス関数(インピーダンス)をCauer形【解法2】で実現せよ。

$$\begin{aligned}
 Z_1(s) &= \frac{(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)} \\
 &= \frac{s^4 + 10s^2 + 9}{s^3 + 4s} \\
 &= \frac{9}{4s} + \frac{\frac{31}{4}s^2 + s^4}{s^3 + 4s} \\
 &= \frac{9}{4s} + \frac{1}{\frac{s^3 + 4s}{\frac{31}{4}s^2 + s^4}} \\
 &= \frac{9}{4s} + \frac{1}{\frac{16}{31s} + \frac{\frac{15}{31}s^3}{\frac{31}{4}s^2 + s^4}} \\
 &= \frac{9}{4s} + \frac{1}{\frac{16}{31s} + \frac{1}{\frac{\frac{31}{4}s^2 + s^4}{\frac{15}{31}s^3}}}
 \end{aligned}$$

$$\begin{array}{r}
 \boxed{\frac{9}{4s}} \\
 \hline
 4s + s^3 \left| \begin{array}{l} 9 + 10s^2 + s^4 \\ 9 + \frac{9}{4}s^2 \end{array} \right. \\
 \hline
 \boxed{\frac{31}{4}s^2 + s^4}
 \end{array}$$

$$\begin{array}{r}
 \boxed{\frac{16}{31s}} \\
 \hline
 \frac{31}{4}s^2 + s^4 \left| \begin{array}{l} 4s + s^3 \\ 4s + \frac{16}{31}s^3 \end{array} \right. \\
 \hline
 \boxed{\frac{15}{31}s^3}
 \end{array}$$

$$= \frac{9}{4s} + \frac{1}{\frac{16}{31s} + \frac{\frac{1}{\frac{31}{4}s^2 + s^4}}{\frac{15}{31}s^3}}$$

$$= \frac{9}{4s} + \frac{1}{\frac{16}{31s} + \frac{1}{\frac{961}{60s} + \frac{s^4}{\frac{15}{31}s^3}}}$$

$$= \frac{9}{4s} + \frac{1}{\frac{16}{31s} + \frac{961}{60s} + \frac{1}{\frac{15}{31}s^3 s^4}}$$

$$= \frac{9}{4s} + \frac{1}{\frac{16}{31s} + \frac{961}{60s} + \frac{1}{\frac{15}{31}s}}$$

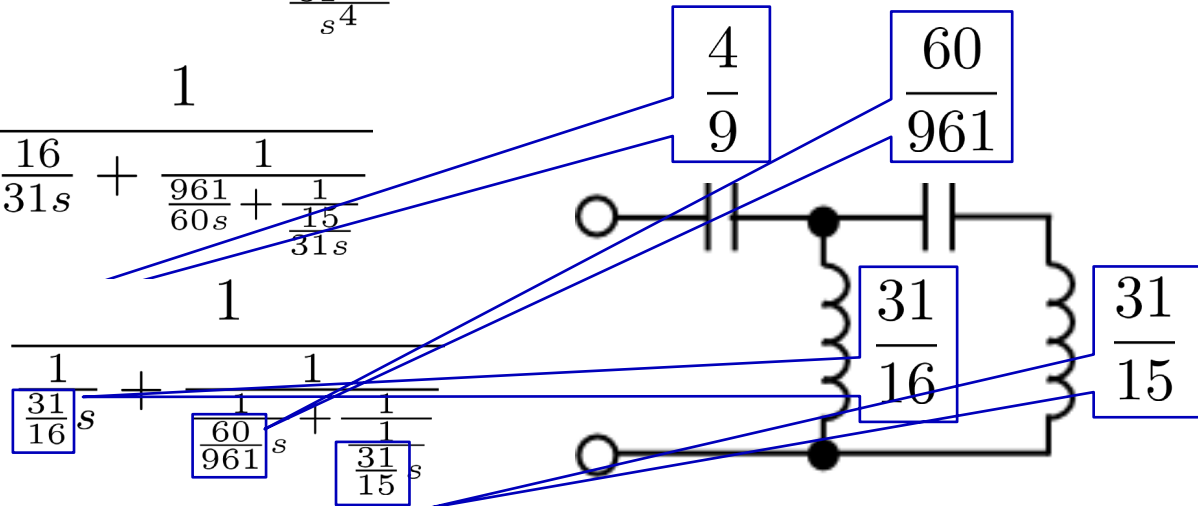
$$= \frac{1}{\frac{4}{9}s} + \frac{1}{\frac{31}{16}s + \frac{60}{961}s + \frac{1}{\frac{31}{15}s}}$$

$$\frac{15}{31}s^3 \sqrt{\frac{\frac{961}{60s}}{\frac{31}{4}s^2 + s^4}}$$

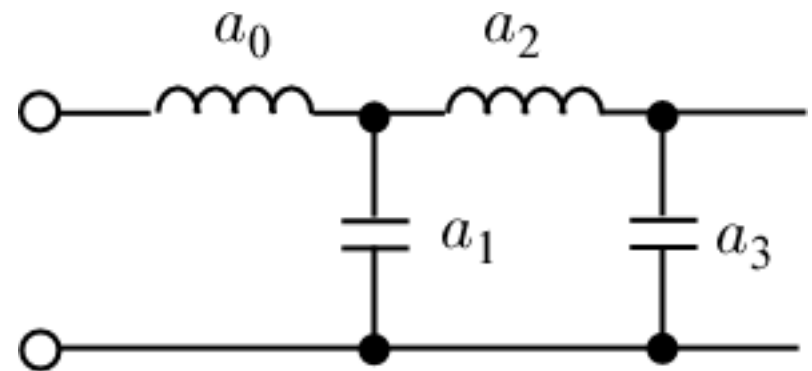
$$\frac{31}{4}s^2$$


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$$s^4$$



$$Z(s) = a_0s + \frac{1}{a_1s + \frac{1}{a_2s + \frac{1}{a_3s + \dots}}}$$

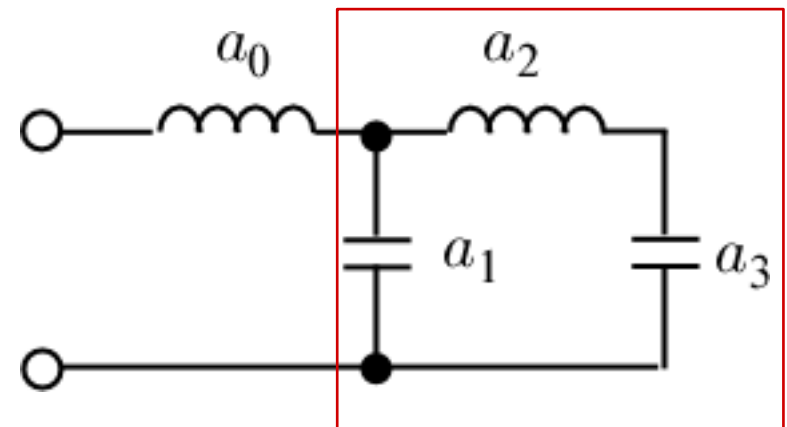


【例】

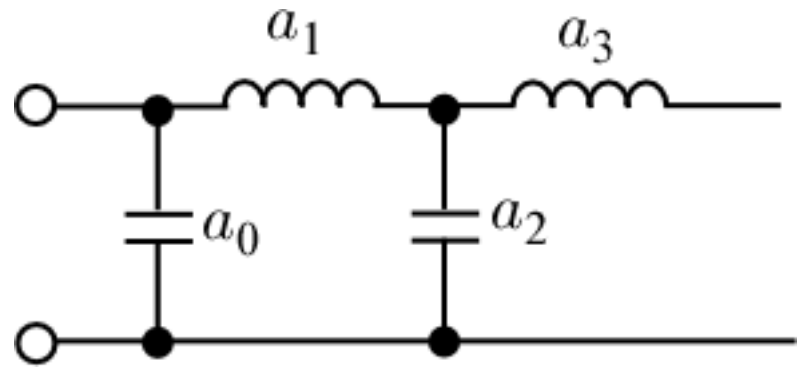
$$Z(s) = a_0s + \frac{\left(a_2s + \frac{1}{a_3s}\right) \frac{1}{a_1s}}{\left(a_2s + \frac{1}{a_3s}\right) + \frac{1}{a_1s}}$$

分母分子を  $\left(a_2s + \frac{1}{a_3s}\right) \frac{1}{a_1s}$  で割る

$$Z(s) = a_0s + \frac{1}{a_1s + \frac{1}{\left(a_2s + \frac{1}{a_3s}\right)}}$$



$$Y(s) = a_0s + \frac{1}{a_1s + \frac{1}{a_2s + \frac{1}{a_3s + \dots}}}$$

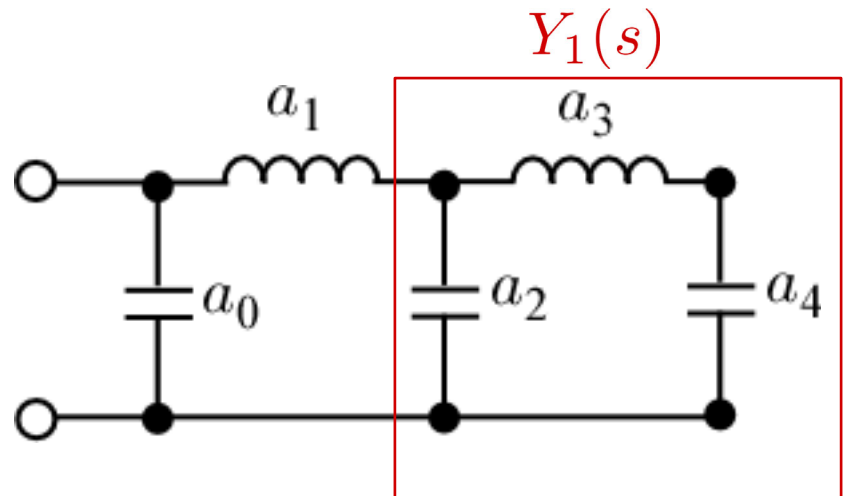


**【例】**

$$Y_1(s) = a_2s + \frac{1}{a_3s + \frac{1}{a_4s}}$$

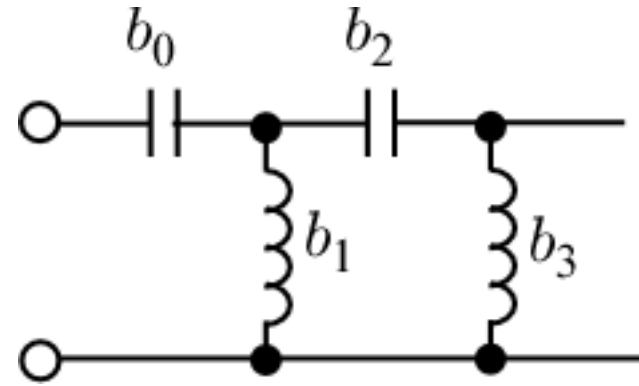
$$Y(s) = a_0s + \frac{1}{a_1s + \frac{1}{Y_1}}$$

$$= a_0s + \frac{1}{a_1s + \frac{1}{a_2s + \frac{1}{a_3s + \frac{1}{a_4s}}}}$$





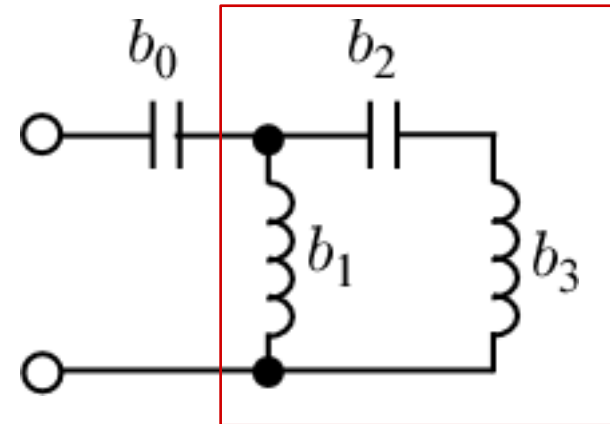
$$Z(s) = \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\frac{b_2}{s} + \frac{1}{\frac{b_3}{s} + \dots}}}$$



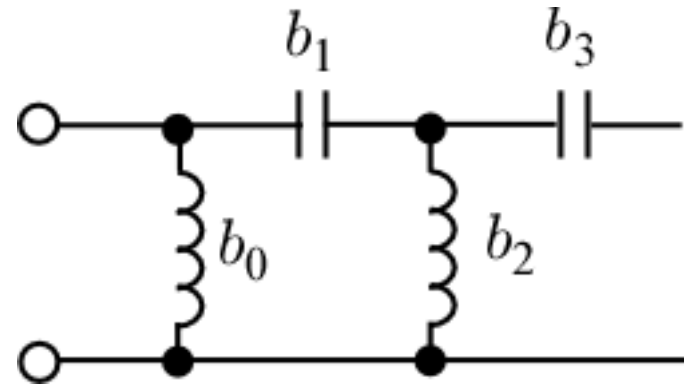
$$C_{2n} = \frac{1}{b_{2n}} \quad L_{n+1} = \frac{1}{b_{n+1}}$$

**【例】**

$$\begin{aligned} Z(s) &= \frac{b_0}{s} + \frac{\left(\frac{b_2}{s} + \frac{s}{b_3}\right) \frac{s}{b_1}}{\left(\frac{b_2}{s} + \frac{s}{b_3}\right) + \frac{s}{b_1}} \\ &= \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\left(\frac{b_2}{s} + \frac{s}{b_3}\right)}} \\ &= \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\frac{b_2}{s} + \frac{1}{\frac{b_3}{s}}}} \end{aligned}$$

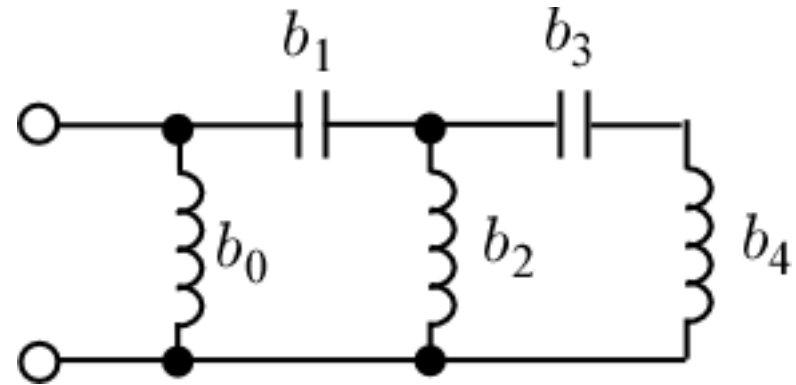


$$Y(s) = \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\frac{b_2}{s} + \frac{1}{\frac{b_3}{s} + \dots}}}$$



【例】

$$\begin{aligned} Y(s) &= \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{Y_1}} \\ &= \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\frac{b_2}{s} + \frac{1}{\frac{b_3}{s} + \frac{s}{b_4}}}} \\ &= \frac{b_0}{s} + \frac{1}{\frac{b_1}{s} + \frac{1}{\frac{b_2}{s} + \frac{1}{\frac{b_3}{s} + \frac{1}{s}}}} \end{aligned}$$



$$Y_1(s) = \frac{b_2}{s} + \frac{1}{\frac{b_3}{s} + \frac{s}{b_4}}$$

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