第 1 章：1端子対回路

1.6 RL 1端子対回路

1.7 RC 1端子対回路

キーワード： Foster展開，Cauer展開

学習目標：RL回路，RC回路をFoster展開やCauer展開で合成することができる。

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1 1端子対回路

1.6 RL 1端子対回路

LC直列回路

$$Z_{LC}(s) = \frac{h_0}{s} + \sum_{k=1}^{n} \frac{h_{2k}s}{s^2+\omega_{2k}^2} + h_\infty s$$

RL直列回路

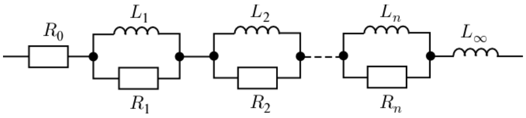
$$Z_{RL}(s) = R + sL$$

$$Z_{RL}(s) = h_0 + \sum_{k=1}^{n} h_k \boxed{-\sum_{k=1}^{n} \frac{\omega_k h_k}{\boxed{s+\omega_k}}} + \boxed{h_\infty s}$$

- 極が負の実軸上および無限遠点にのみ存在
  $Z_{RL}(s) = \infty$ となるとき $s = -\omega_k,\ \infty$
- 無限遠点を除く極の留数は負の値をとる
  $$[(s+\omega_k)Z_{RL}(s)]_{s=-\omega_k} = -\sum_{k=1}^{n} \omega_k h_k$$

Foster形

$$Z_{RL}(s) = h_0 + \sum_{k=1}^{n} \frac{h_k s}{s+\omega_k} + h_\infty s$$



$$R_0 = h_0 = [Z_{RL}(s)]_{s=0}$$

$$L_\infty = \left[\frac{Z_{RL}(s)}{s}\right]_{s=\infty}$$

$$R_k = h_k = \left[\frac{s+\omega_k}{s} Z_{RL}(s)\right]_{s=-\omega_k}$$

$$L_k = \frac{R_k}{\omega_k} = \frac{1}{\omega_k}\left[\frac{s+\omega_k}{s} Z_{RL}(s)\right]_{s=-\omega_k}$$

Cauer形

$$Z_{RL}(s) = h_0 + \sum_{k=1}^{n} \frac{h_k s}{s+\omega_k} + h_\infty s$$

$$= a_0 s + \cfrac{1}{a_1 + \cfrac{1}{a_2 s + \cfrac{1}{a_3 + \cdots}}}$$

$$= b_0 + \cfrac{1}{\frac{b_1}{s} + \cfrac{1}{b_2 + \cfrac{1}{\frac{b_3}{s} + \cdots}}}$$

$$= H\frac{s(s+\omega_1)(s+\omega_3)\cdots(s+\omega_{2n+1})}{(s+\omega_2)\cdots(s+\omega_{2n})}$$

RC直列回路

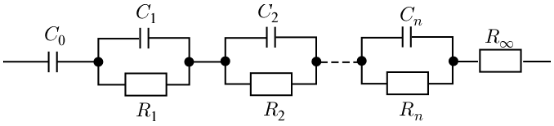
$$Z_{RC}(s) = R + \frac{1}{sC}$$

$$Z_{RC}(s) = \left[\frac{1}{s}Z_{LC}(s)\right]_{s^2=s} = \frac{\boxed{h_0}}{s} + \sum_{k=1}^{n} \frac{\boxed{h_k}}{\boxed{s+\omega_k}} + h_\infty$$

- $s=0$ の点を含むすべての極が負の実軸上に存在し，その留数は正
  $$[(s+\omega_k)Z_{RC}(s)]_{s=-\omega_k} = \sum_{k=1}^{n} h_k$$
  $$[sZ_{RC}(s)]_{s=0} = h_0$$
- $s=\infty$ の点は零点または有限な実数値であり，負の実軸上の最右端には極が存在する

Foster形

$$Z_{RC}(s) = \frac{h_0}{s} + \sum_{k=1}^{n} \frac{h_k}{s+\omega_k} + h_\infty$$



$$C_0 = \frac{1}{h_0} = \frac{1}{[sZ_{RC}(s)]_{s=0}}$$

$$R_\infty = h_\infty = [Z_{RC}(s)]_{s=\infty}$$

$$C_k = \frac{1}{h_k} = \frac{1}{[(s+\omega_k)Z_{RL}(s)]_{s=-\omega_k}}$$

$$R_k = \frac{h_k}{\omega_k} = \frac{[(s+\omega_k)Z_{RL}(s)]_{s=-\omega_k}}{\omega_k}$$

Cauer形

$$Z_{RC}(s)=\frac{h_0}{s}+\sum_{k=1}^{n}\frac{h_k}{s+\omega_k}+h_\infty$$
$$=a_0+\cfrac{1}{a_1s+\cfrac{1}{a_2+\cfrac{1}{a_3s+\cdots}}}$$
$$=\frac{b_0}{s}+\cfrac{1}{b_1+\cfrac{1}{\frac{b_2}{s}+\cfrac{1}{b_3+\cdots}}}$$
$$=H\frac{(s+\omega_1)(s+\omega_3)\cdots(s+\omega_{2n+1})}{s(s+\omega_2)\cdots(s+\omega_{2n})}$$

【例】第1章【23】(1)

インピーダンス関数をFoster形で実現せよ。

$$Z_1(s)=\frac{(s+3)(s+5)}{(s+2)(s+4)},\qquad Y_1(s)=\frac{1}{Z_1(s)}$$

(a) インピーダンスの場合

極の留数は正の値よりRL回路でなく，RC回路

$$[(s+2)Z_1(s)]_{s=-2}=\frac{(-2+3)(-2+5)}{-2+4}>0$$
$$[(s+4)Z_1(s)]_{s=-4}=\frac{(-4+3)(-4+5)}{-4+2}>0$$

$$Z_1(s)=\frac{h_0}{s}+\frac{h_1}{s+2}+\frac{h_2}{s+4}+h_\infty$$

$$h_0=[sZ_1(s)]_{s=0}=0 \qquad Z_1(s)=\frac{(s+3)(s+5)}{(s+2)(s+4)},$$

$$h_\infty=[Z_1(s)]_{s=\infty}=\left[\frac{s^2}{s^2}\right]_{s=\infty}=1$$

$$h_1=[(s+2)Z_1(s)]_{s=-2}=\left[\frac{(s+3)(s+5)}{s+4}\right]_{s=-2}=\frac{1\times 3}{2}=\frac{3}{2}$$

$$h_2=[(s+4)Z_1(s)]_{s=-4}=\left[\frac{(s+3)(s+5)}{s+2}\right]_{s=-4}=\frac{-1\times 1}{-2}=\frac{1}{2}$$

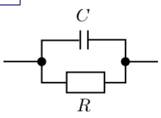
$$Z_1(s)=\frac{\frac{3}{2}}{s+2}+\frac{\frac{1}{2}}{s+4}+1$$

$$Z_1(s)=\frac{\overset{\frac{1}{C_1}}{\boxed{\frac{3}{2}}}}{s+\underset{\frac{1}{R_1C_1}}{\boxed{2}}}+\frac{\overset{\frac{1}{C_2}}{\boxed{\frac{1}{2}}}}{s+\underset{\frac{1}{R_2C_2}}{\boxed{4}}}+1$$

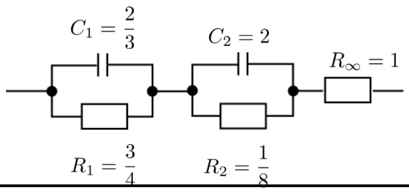
$$=\frac{1}{\underset{C_1}{\boxed{\frac{2}{3}}}s+\underset{\frac{1}{R_1}}{\boxed{\frac{4}{3}}}}+\frac{1}{2s+8}+1$$

$$Z(s)=\frac{\frac{R}{sC}}{R+\frac{1}{sC}}=\frac{R}{sRC+1}$$
$$=\frac{\frac{1}{C}}{s+\frac{1}{RC}}$$

$$Z=\frac{1}{sC+\frac{1}{R}}$$



上記のどちらで考えてもよい



(b) アドミタンスの場合

$$Y_1(s)=\frac{1}{Z_1(s)}=\frac{(s+2)(s+4)}{(s+3)(s+5)}$$

$$Y_1(s)=h_0+\frac{h_1s}{s+3}+\frac{h_2s}{s+5}+h_\infty s$$

$$h_0=[Y_1(s)]_{s=0}=\frac{2\times 4}{3\times 5}=\frac{8}{15}$$

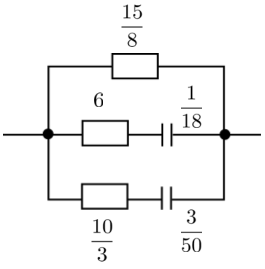
$$h_\infty=\left[\frac{1}{s}Y_1(s)\right]_{s=\infty}=0$$

$$h_1=\left[\frac{s+3}{s}Y_1(s)\right]_{s=-3}=\frac{-1\times 1}{-3\times 2}=\frac{1}{6}$$

$$h_2=\left[\frac{s+5}{s}Y_1(s)\right]_{s=-5}=\frac{-3\times(-1)}{-5\times(-2)}=\frac{3}{10}$$

$$Y_1(s)=\frac{8}{15}+\frac{\frac{1}{6}s}{s+3}+\frac{\frac{3}{10}s}{s+5}$$

$$Y_1(s)=\frac{8}{15}+\frac{\frac{1}{6}s}{s+3}+\frac{\frac{3}{10}s}{s+5}$$
$$=\frac{8}{15}+\frac{1}{6+\frac{18}{s}}+\frac{1}{\frac{10}{3}+\frac{50}{3s}}$$



【例】第1章【23】(1)特別解答1

インピーダンス関数をCauer形で実現せよ。

$$Z_1(s)=\frac{(s+3)(s+5)}{(s+2)(s+4)}$$

$$=\frac{s^2+8s+15}{s^2+6s+8}$$

$$=\boxed{1}+\frac{\boxed{2s+7}}{s^2+6s+8}=1+\frac{1}{\frac{s^2+6s+8}{2s+7}}$$

$$=1+\frac{1}{\boxed{\frac{1}{2}s}+\frac{\boxed{\frac{5}{2}s+8}}{2s+7}}=1+\frac{1}{\frac{1}{2}s+\frac{1}{\frac{2s+7}{\frac{5}{2}s+8}}}$$

$$=1+\frac{1}{\frac{1}{2}s+\frac{1}{\frac{2s+7}{\frac{5}{2}s+8}}}$$

$$=1+\frac{1}{\frac{1}{2}s+\frac{1}{\boxed{\frac{4}{5}}+\frac{\boxed{\frac{3}{5}}}{\frac{5}{2}s+8}}}$$

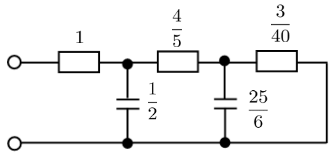
$$\begin{array}{r|l} & \boxed{1} \\ \hline s^2+6s+8 & s^2+8s+15 \\ & s^2+6s+8 \\ \hline & \boxed{2s+7} \end{array}$$

$$\begin{array}{r|l} & \boxed{\frac{1}{2}s} \\ \hline 2s+7 & s^2+6s+8 \\ & s^2+\frac{7}{2}s \\ \hline & \boxed{\frac{5}{2}s+8} \end{array}$$

$$\begin{array}{r|l} & \boxed{\frac{4}{5}} \\ \hline \frac{5}{2}s+8 & 2s+7 \\ & 2s+\frac{32}{5} \\ \hline & \boxed{\frac{3}{5}} \end{array}$$

$$=1+\frac{1}{\frac{1}{2}s+\frac{1}{\frac{4}{5}+\frac{\frac{3}{5}}{\frac{5}{2}s+8}}}=1+\frac{1}{\frac{1}{2}s+\frac{1}{\frac{4}{5}+\frac{1}{\frac{25}{6}s+\frac{40}{3}}}}$$

$$=1+\frac{1}{\frac{1}{2}s+\frac{1}{\frac{4}{5}+\frac{1}{\frac{25}{6}s+\frac{1}{\frac{3}{40}}}}}$$



【例】第1章【23】(1)特別解答2

インピーダンス関数をCauer形で実現せよ。

$$Z_1(s)=\frac{(s+3)(s+5)}{(s+2)(s+4)}$$

$$=\frac{s^2+8s+15}{s^2+6s+8}$$

$$\begin{array}{r|l} & \boxed{\frac{15}{8}} \\ \hline 8+6s+s^2 & 15+8s+s^2 \\ & 15+\frac{90}{8}s+\frac{15}{8}s^2 \\ \hline & \boxed{\boxed{-}\frac{26}{8}s\boxed{-}\frac{7}{8}s^2} \end{array}$$

実現できない

$$Z_1(s)=\frac{1}{\frac{s^2+6s+8}{s^2+8s+15}}$$

$$=\frac{1}{\boxed{\frac{8}{15}}+\frac{\boxed{\frac{26}{15}s+\frac{7}{15}s^2}}{s^2+8s+15}}$$

$$\begin{array}{r|l} & \boxed{\frac{8}{15}} \\ \hline 15+8s+s^2 & 8+6s+s^2 \\ & 8+\frac{64}{15}s+\frac{8}{15}s^2 \\ \hline & \boxed{\frac{26}{15}s+\frac{7}{15}s^2} \end{array}$$

$$=\frac{1}{\frac{8}{15}+\frac{\frac{26}{15}s+\frac{7}{15}s^2}{s^2+8s+15}}=\frac{1}{\frac{8}{15}+\frac{1}{\frac{s^2+8s+15}{\frac{26}{15}s+\frac{7}{15}s^2}}}$$

$$=\frac{1}{\frac{8}{15}+\frac{1}{\boxed{\frac{15^2}{26s}}+\frac{\boxed{\frac{103}{26}s+s^2}}{\frac{26}{15}s+\frac{7}{15}s^2}}}$$

$$=\frac{1}{\frac{8}{15}+\frac{1}{\frac{15^2}{26s}+\frac{1}{\frac{\frac{26}{15}s+\frac{7}{15}s^2}{\frac{103}{26}s+s^2}}}}$$

$$=\frac{1}{\frac{8}{15}+\frac{1}{\frac{15^2}{26s}+\frac{1}{\frac{26^2}{103\times 15}+\frac{\frac{45}{103\times 15}s^2}{\frac{103}{26}s+s^2}}}}$$

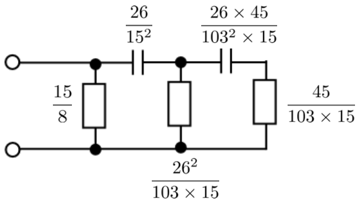
$$=\frac{1}{\frac{8}{15}+\frac{1}{\frac{15^2}{26s}+\frac{1}{\frac{26^2}{103\times 15}+\frac{1}{\frac{\frac{103}{26}s+s^2}{\frac{45}{103\times 15}s^2}}}}}$$

$$\begin{array}{r|l} & \boxed{\frac{15^2}{26s}} \\ \hline \frac{26}{15}s+\frac{7}{15}s^2 & 15+8s+s^2 \\ & 15+\frac{105}{26}s \\ \hline & \boxed{\frac{103}{26}s+s^2} \end{array}$$

$$\begin{array}{r|l} & \boxed{\frac{26^2}{103\times 15}} \\ \hline \frac{103}{26}s+s^2 & \frac{26}{15}s+\frac{7}{15}s^2 \\ & \frac{26}{15}s+\frac{26^2}{103\times 15}s^2 \\ \hline & \boxed{\frac{45}{103\times 15}s^2} \end{array}$$

$$=\frac{1}{\frac{8}{15}+\frac{1}{\frac{15^2}{26s}+\frac{1}{\frac{26^2}{103\times 15}+\frac{1}{\frac{\frac{103}{26}s+s^2}{\frac{45}{103\times 15}s^2}}}}}$$

$$=\frac{1}{\frac{8}{15}+\frac{1}{\frac{15^2}{26s}+\frac{1}{\frac{26^2}{103\times 15}+\frac{1}{\frac{103^2\times 15}{26\times 45s}+\frac{103\times 15}{45}}}}}$$



第 1 章：1端子対回路

1.6 *RL* 1端子対回路

1.7 *RC* 1端子対回路

キーワード： Foster展開，Cauer展開

学習目標：*RL*回路，*RC*回路をFoster展開やCauer展開で合成することができる。