

第1章：1端子対回路

1.6 RL 1端子対回路

1.7 RC 1端子対回路

キーワード： Foster展開, Cauer展開

学習目標： RL 回路, RC 回路をFoster展開やCauer展開で合成することができる。

1 1端子対回路

1.6 RL 1端子対回路

RL直列回路

$$Z_{RL}(s) = R + sL$$

$$Z_{RL}(s) = h_0 + \sum_{k=1}^n h_k - \sum_{k=1}^n \frac{\omega_k h_k}{s + \omega_k} + h_\infty s$$

- 極が負の実軸上および無限遠点にのみ存在

$$Z_{RL}(s) = \infty \text{ となるとき } s = -\omega_k, \infty$$

- 無限遠点を除く極の留数は負の値をとる

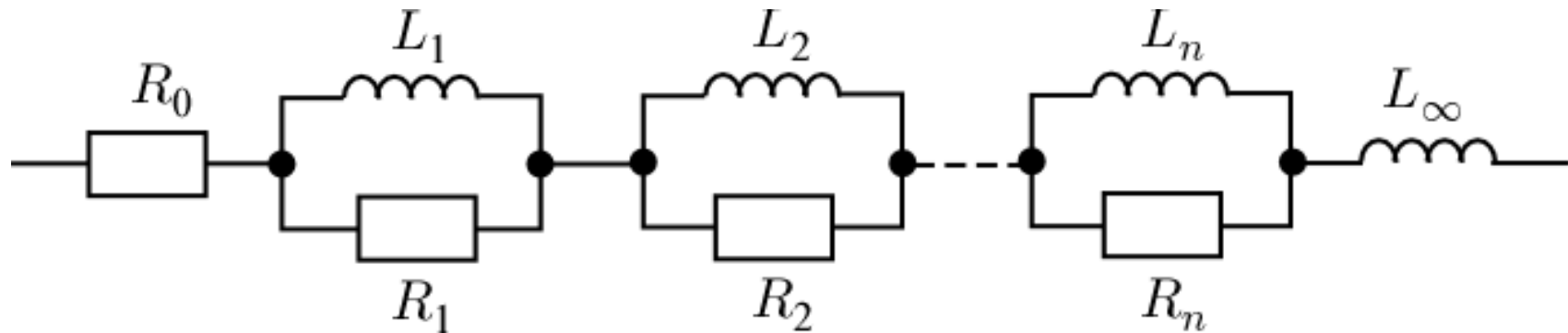
$$[(s + \omega_k)Z_{RL}(s)]_{s=-\omega_k} = -\sum_{k=1}^n \omega_k h_k$$

LC直列回路

$$Z_{LC}(s) = \frac{h_0}{s} + \sum_{k=1}^n \frac{h_{2k}s}{s^2 + \omega_{2k}^2} + h_\infty s$$

Foster形

$$Z_{RL}(s) = h_0 + \sum_{k=1}^n \frac{h_k s}{s + \omega_k} + h_\infty s$$



$$R_0 = h_0 = [Z_{RL}(s)]_{s=0}$$

$$L_\infty = \left[\frac{Z_{RL}(s)}{s} \right]_{s=\infty}$$

$$R_k = h_k = \left[\frac{s + \omega_k}{s} Z_{RL}(s) \right]_{s=-\omega_k}$$

$$L_k = \frac{R_k}{\omega_k} = \frac{1}{\omega_k} \left[\frac{s + \omega_k}{s} Z_{RL}(s) \right]_{s=-\omega_k}$$

Cauer形

$$Z_{RL}(s) = h_0 + \sum_{k=1}^n \frac{h_k s}{s + \omega_k} + h_\infty s$$

$$= a_0 s + \frac{1}{a_1 + \frac{1}{a_2 s + \frac{1}{a_3 + \dots}}}$$

$$= b_0 + \frac{1}{\frac{b_1}{s} + \frac{1}{b_2 + \frac{1}{\frac{b_3}{s} + \dots}}}$$

$$= H \frac{s(s + \omega_1)(s + \omega_3) \cdots (s + \omega_{2n+1})}{(s + \omega_2) \cdots (s + \omega_{2n})}$$

RC直列回路

$$Z_{RC}(s) = R + \frac{1}{sC}$$

$$Z_{RC}(s) = \left[\frac{1}{s} Z_{LC}(s) \right]_{s^2=s} = \frac{h_0}{s} + \sum_{k=1}^n \frac{h_k}{s + \omega_k} + h_\infty$$

- $s = 0$ の点を含むすべての極が負の実軸上に存在し、
その留数は正

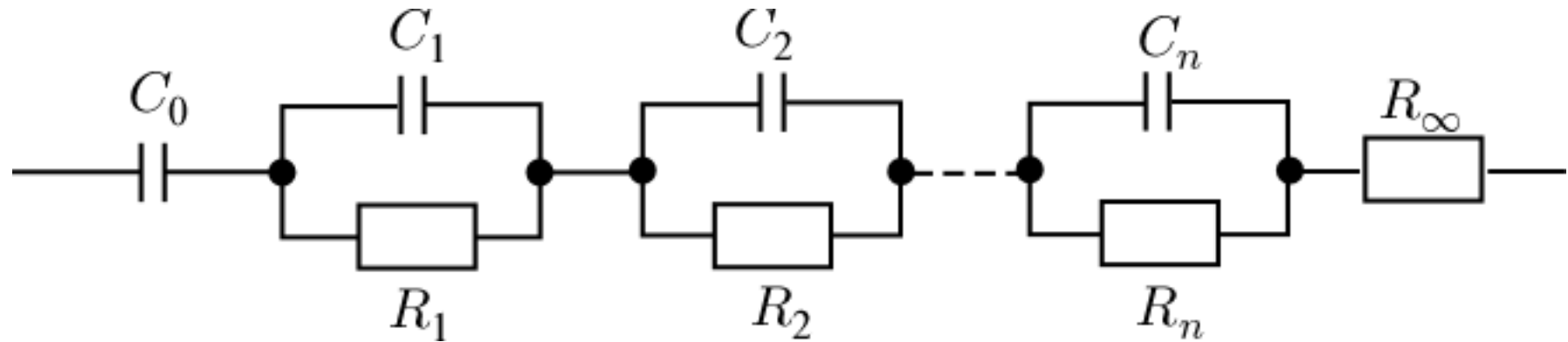
$$[(s + \omega_k) Z_{RC}(s)]_{s=-\omega_k} = \sum_{k=1}^n h_k$$

$$[s Z_{RC}(s)]_{s=0} = h_0$$

- $s = \infty$ の点は零点または有限な実数値であり、負の実軸上の最右端には極が存在する

Foster形

$$Z_{RC}(s) = \frac{h_0}{s} + \sum_{k=1}^n \frac{h_k}{s + \omega_k} + h_\infty$$



$$C_0 = \frac{1}{h_0} = \frac{1}{[sZ_{RC}(s)]_{s=0}}$$

$$R_\infty = h_\infty = [Z_{RC}(s)]_{s=\infty}$$

$$C_k = \frac{1}{h_k} = \frac{1}{[(s + \omega_k)Z_{RL}(s)]_{s=-\omega_k}}$$

$$R_k = \frac{h_k}{\omega_k} = \frac{[(s + \omega_k)Z_{RL}(s)]_{s=-\omega_k}}{\omega_k}$$

Cauer形

$$\begin{aligned} Z_{RC}(s) &= \frac{h_0}{s} + \sum_{k=1}^n \frac{h_k}{s + \omega_k} + h_\infty \\ &= a_0 + \frac{1}{a_1 s + \frac{1}{a_2 + \frac{1}{a_3 s + \dots}}} \\ &= \frac{b_0}{s} + \frac{1}{b_1 + \frac{1}{\frac{b_2}{s} + \frac{1}{b_3 + \dots}}} \\ &= H \frac{(s + \omega_1)(s + \omega_3) \cdots (s + \omega_{2n+1})}{s(s + \omega_2) \cdots (s + \omega_{2n})} \end{aligned}$$

【例】第1章【23】(1)

インピーダンス関数をFoster形で実現せよ。

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)}, \quad Y_1(s) = \frac{1}{Z_1(s)}$$

(a) インピーダンスの場合

極の留数は正の値よりRL回路でなく, RC回路

$$[(s+2)Z_1(s)]_{s=-2} = \frac{(-2+3)(-2+5)}{-2+4} > 0$$

$$[(s+4)Z_1(s)]_{s=-4} = \frac{(-4+3)(-4+5)}{-4+2} > 0$$

$$Z_1(s) = \frac{h_0}{s} + \frac{h_1}{s+2} + \frac{h_2}{s+4} + h_\infty$$

$$h_0 = [sZ_1(s)]_{s=0} = 0 \qquad Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)},$$

$$h_\infty = [Z_1(s)]_{s=\infty} = \left[\frac{s^2}{s^2} \right]_{s=\infty} = 1$$

$$h_1 = [(s+2)Z_1(s)]_{s=-2} = \left[\frac{(s+3)(s+5)}{s+4} \right]_{s=-2} = \frac{1 \times 3}{2} = \frac{3}{2}$$

$$h_2 = [(s+4)Z_1(s)]_{s=-4} = \left[\frac{(s+3)(s+5)}{s+2} \right]_{s=-4} = \frac{-1 \times 1}{-2} = \frac{1}{2}$$

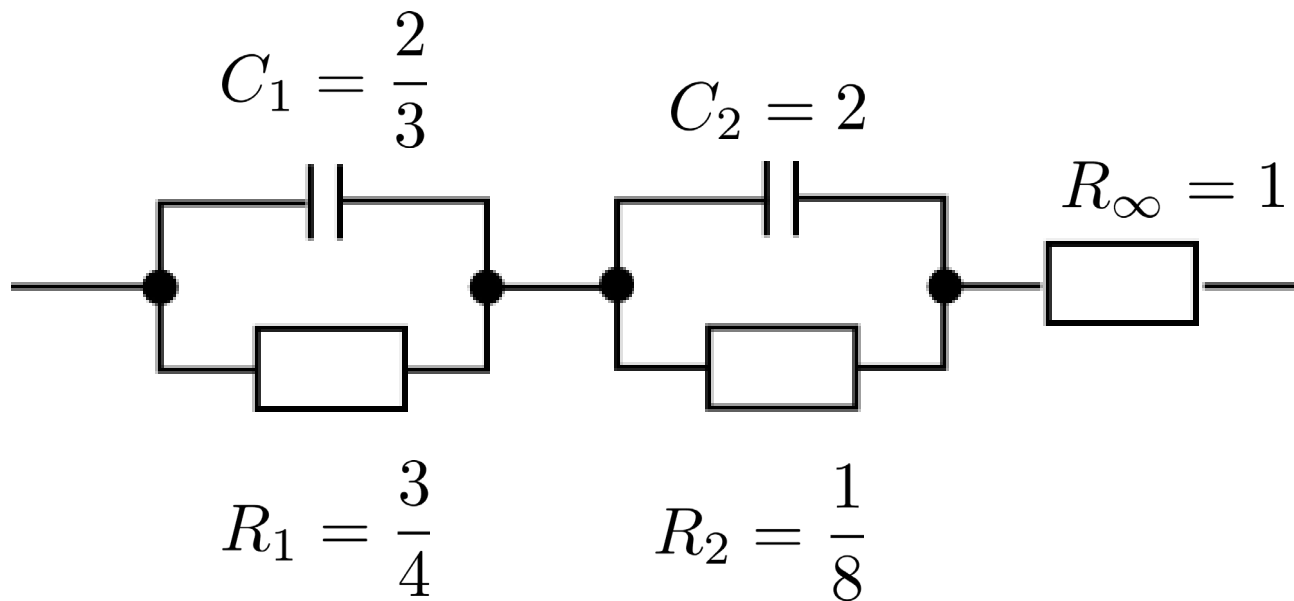
$$Z_1(s) = \frac{\frac{3}{2}}{s+2} + \frac{\frac{1}{2}}{s+4} + 1$$

$$Z_1(s) = \frac{\frac{1}{C_1}}{s + \frac{1}{R_1 C_1}} + \frac{\frac{1}{C_2}}{s + \frac{1}{R_2 C_2}} + 1$$

$$= \frac{1}{\frac{2}{3}s + \frac{4}{3}} + \frac{1}{2s + 8} + 1$$

C_1 $\frac{1}{R_1}$

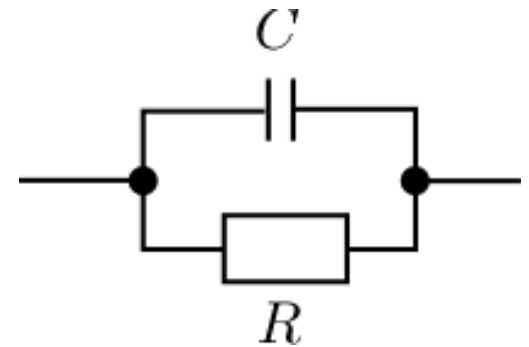
上記のどちらで考えてもよい



$$Z(s) = \frac{\frac{R}{sC}}{R + \frac{1}{sC}} = \frac{R}{sRC + 1}$$

$$= \frac{\frac{1}{C}}{s + \frac{1}{RC}}$$

$$Z = \frac{1}{sC + \frac{1}{R}}$$



(b) アドミタンスの場合

$$Y_1(s) = \frac{1}{Z_1(s)} = \frac{(s+2)(s+4)}{(s+3)(s+5)}$$

$$Y_1(s) = h_0 + \frac{h_1 s}{s+3} + \frac{h_2 s}{s+5} + h_\infty s$$

$$h_0 = [Y_1(s)]_{s=0} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

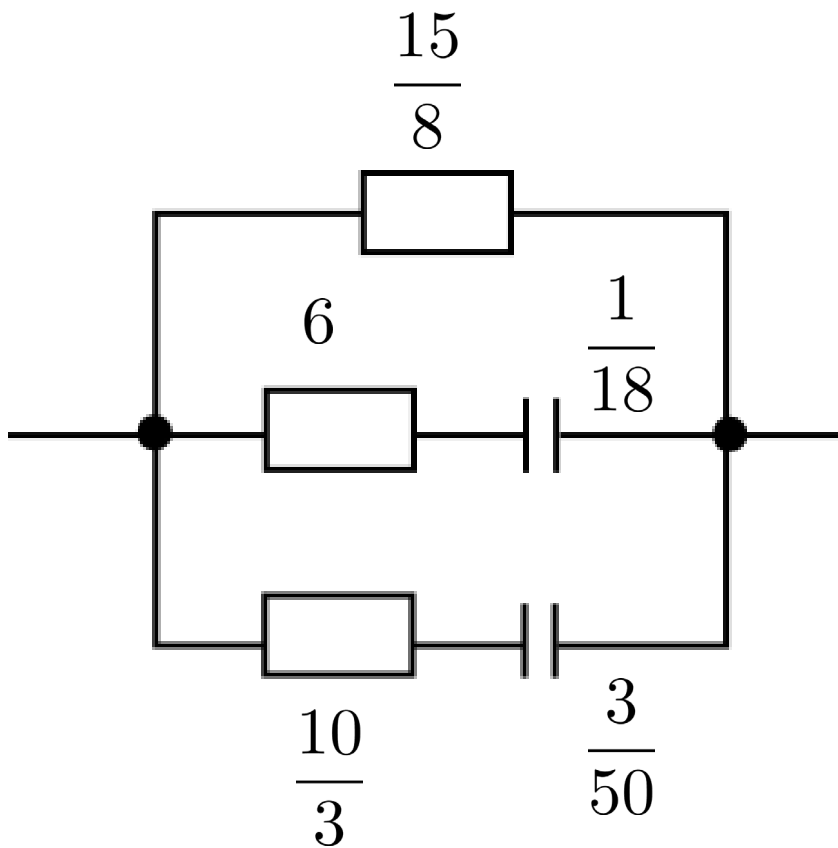
$$h_\infty = \left[\frac{1}{s} Y_1(s) \right]_{s=\infty} = 0$$

$$h_1 = \left[\frac{s+3}{s} Y_1(s) \right]_{s=-3} = \frac{-1 \times 1}{-3 \times 2} = \frac{1}{6}$$

$$h_2 = \left[\frac{s+5}{s} Y_1(s) \right]_{s=-5} = \frac{-3 \times (-1)}{-5 \times (-2)} = \frac{3}{10}$$

$$Y_1(s) = \frac{8}{15} + \frac{\frac{1}{6}s}{s+3} + \frac{\frac{3}{10}s}{s+5}$$

$$\begin{aligned}
 Y_1(s) &= \frac{8}{15} + \frac{\frac{1}{6}s}{s+3} + \frac{\frac{3}{10}s}{s+5} \\
 &= \frac{8}{15} + \frac{1}{6 + \frac{18}{s}} + \frac{1}{\frac{10}{3} + \frac{50}{3s}}
 \end{aligned}$$



【例】第1章【23】(1)特別解答1

インピーダンス関数をCauer形で実現せよ。

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)}$$

$$= \frac{s^2 + 8s + 15}{s^2 + 6s + 8}$$

$$= \boxed{1} + \frac{\boxed{2s+7}}{s^2 + 6s + 8} = 1 + \frac{1}{\frac{s^2+6s+8}{2s+7}}$$

$$= 1 + \frac{1}{\boxed{\frac{1}{2}s} + \frac{\boxed{\frac{5}{2}s+8}}{2s+7}} = 1 + \frac{1}{\frac{1}{2}s + \frac{1}{\frac{5}{2}s+8}}$$

$$= 1 + \frac{1}{\frac{1}{2}s + \frac{1}{\frac{5}{2}s+8}}$$

$$= 1 + \frac{1}{\frac{1}{2}s + \frac{\boxed{\frac{4}{5}}}{\frac{5}{2}s+8} + \frac{\boxed{\frac{3}{5}}}{5}}$$

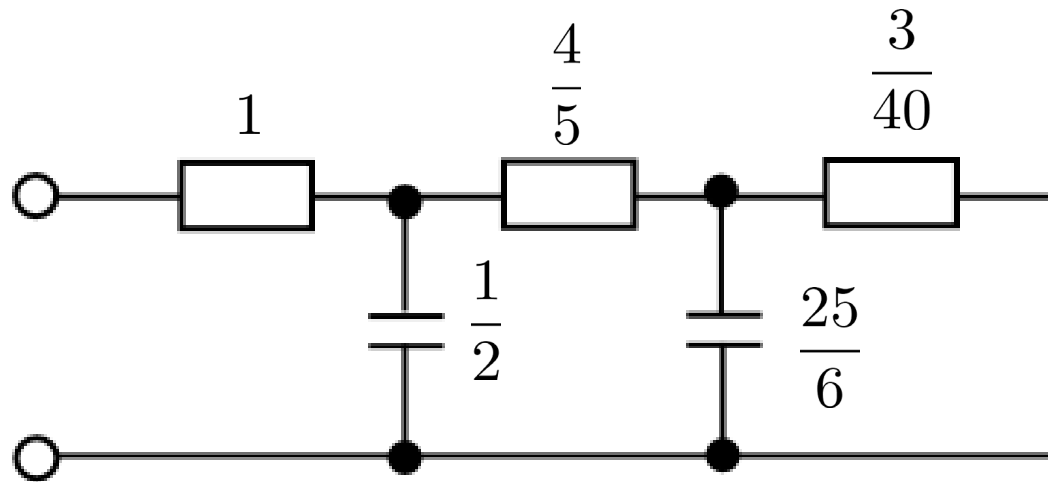
$$s^2 + 6s + 8 \overline{) \begin{array}{r} \boxed{1} \\ s^2 + 8s + 15 \\ \hline s^2 + 6s + 8 \\ \hline \boxed{2s + 7} \end{array}}$$

$$2s + 7 \overline{) \begin{array}{r} \boxed{\frac{1}{2}s} \\ s^2 + 6s + 8 \\ \hline s^2 + \frac{7}{2}s \\ \hline \boxed{\frac{5}{2}s + 8} \end{array}}$$

$$\frac{5}{2}s + 8 \overline{) \begin{array}{r} \boxed{\frac{4}{5}} \\ 2s + 7 \\ \hline 2s + \frac{32}{5} \\ \hline \boxed{\frac{3}{5}} \end{array}}$$

$$= 1 + \frac{1}{\frac{1}{2}s + \frac{1}{\frac{4}{5} + \frac{3}{\frac{5}{2}(s+8)}}} = 1 + \frac{1}{\frac{1}{2}s + \frac{4}{5} + \frac{1}{\frac{25}{6}s + \frac{40}{3}}}$$

$$= 1 + \frac{1}{\frac{1}{2}s + \frac{4}{5} + \frac{1}{\frac{25}{6}s + \frac{1}{\frac{3}{40}}}}$$



【例】第1章【23】(1)特別解答2

インピーダンス関数をCauer形で実現せよ。

$$Z_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)}$$

$$= \frac{s^2 + 8s + 15}{s^2 + 6s + 8}$$

$$8 + 6s + s^2 \overline{) \begin{array}{l} \boxed{\frac{15}{8}} \\ 15 + 8s + s^2 \\ \hline 15 + \frac{90}{8}s + \frac{15}{8}s^2 \\ \hline \boxed{-\frac{26}{8}}s \quad \boxed{-\frac{7}{8}}s^2 \end{array}}$$

実現できない

$$Z_1(s) = \frac{1}{\frac{s^2+6s+8}{s^2+8s+15}}$$

$$= \frac{1}{\boxed{\frac{8}{15}} + \frac{\boxed{\frac{26}{15}s + \frac{7}{15}s^2}}{s^2+8s+15}}$$

$$15 + 8s + s^2 \overline{) \begin{array}{l} \boxed{\frac{8}{15}} \\ 8 + 6s + s^2 \\ \hline 8 + \frac{64}{15}s + \frac{8}{15}s^2 \\ \hline \boxed{\frac{26}{15}}s + \frac{7}{15}s^2 \end{array}}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{26}{15}s + \frac{7}{15}s^2}{s^2 + 8s + 15}} = \frac{1}{\frac{8}{15} + \frac{1}{\frac{s^2 + 8s + 15}{\frac{26}{15}s + \frac{7}{15}s^2}}}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{15^2}{26s} + \frac{\frac{103}{26}s + s^2}{\frac{26}{15}s + \frac{7}{15}s^2}}{1}}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{15^2}{26s} + \frac{1}{\frac{\frac{26}{15}s + \frac{7}{15}s^2}{\frac{103}{26}s + s^2}}}{1}}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{15^2}{26s} + \frac{1}{\frac{26^2}{103 \times 15} + \frac{\frac{45}{103 \times 15}s^2}{\frac{103}{26}s + s^2}}}{1}}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{15^2}{26s} + \frac{1}{\frac{26^2}{103 \times 15} + \frac{\frac{103}{26}s + s^2}{\frac{45}{103 \times 15}s^2}}}{1}}$$

$$\frac{26}{15}s + \frac{7}{15}s^2 \sqrt{\frac{\frac{15^2}{26s}}{15 + 8s + s^2}}$$

$$15 + \frac{105}{26}s$$

$$\frac{103}{26}s + s^2$$

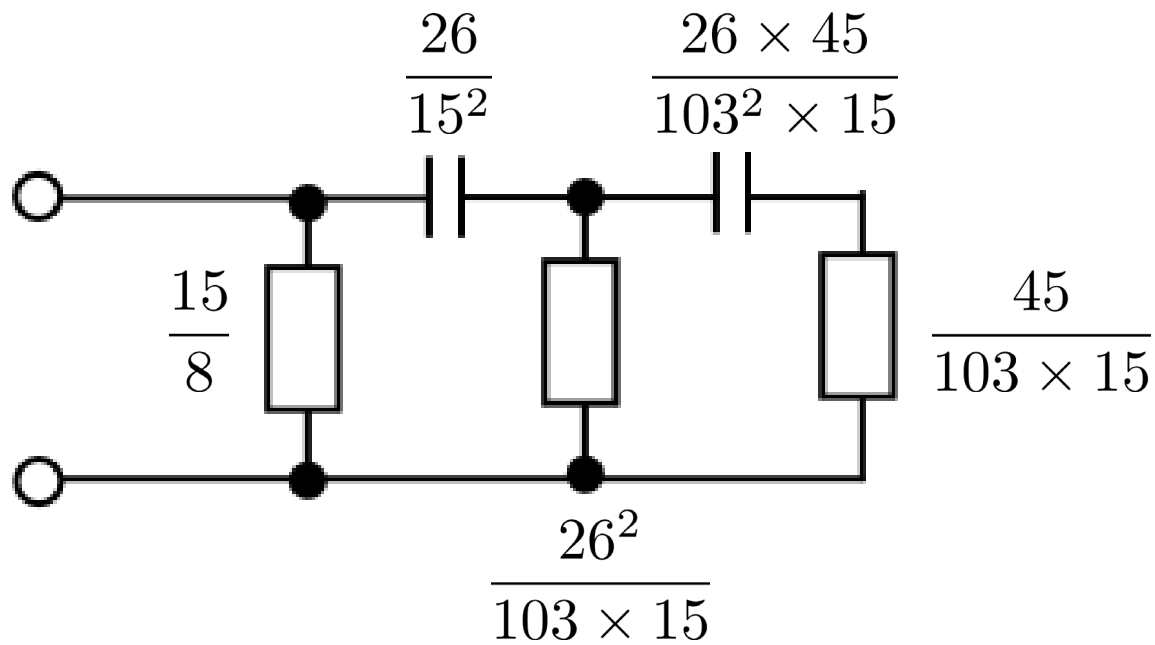
$$\frac{103}{26}s + s^2 \sqrt{\frac{\frac{26^2}{103 \times 15}}{\frac{26}{15}s + \frac{7}{15}s^2}}$$

$$\frac{26}{15}s + \frac{26^2}{103 \times 15}s^2$$

$$\frac{45}{103 \times 15}s^2$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{15^2}{26s} + \frac{1}{\frac{26^2}{103 \times 15} + \frac{1}{\frac{103}{26} \frac{1}{s+s^2} + \frac{45}{103 \times 15} s^2}}}}$$

$$= \frac{1}{\frac{8}{15} + \frac{\frac{15^2}{26s} + \frac{1}{\frac{26^2}{103 \times 15} + \frac{1}{\frac{103^2 \times 15}{26 \times 45s} + \frac{103 \times 15}{45}}}}}}$$



第1章：1端子対回路

1.6 RL 1端子対回路

1.7 RC 1端子対回路

キーワード： Foster展開, Cauer展開

学習目標： RL 回路, RC 回路をFoster展開やCauer展開で合成することができる。