

第 24 章 : 非正弦波交流

24.4 ひずみ波の実効値

24.5 ひずみ率

キーワード : ひずみ波の実効値, ひずみ率

学習目標 : ひずみ波の実効値, ひずみ率を計算できる。  
ひずみ波の交流電力を求めることができる。

1

6 ひずみ波

6.4 ひずみ波の実効値

$$i(t) = I_0 + I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots$$

$$|I| = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} \quad \omega = \frac{2\pi}{T}$$

$$|I| = \sqrt{I_0^2 + |I_1|^2 + |I_2|^2 + \dots}$$

証明

$$\begin{aligned} i(t)^2 &= (I_0 + I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &\quad \times (I_0 + I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &= (I_0^2 + I_{m1}^2 \sin^2(\omega t + \theta_1) + I_{m2}^2 \sin^2(2\omega t + \theta_2) + \dots) \\ &\quad + 2I_0 (I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &\quad + 2I_{m1} \sin(\omega t + \theta_1) (I_{m2} \sin(2\omega t + \theta_2) + \dots) \end{aligned}$$

2

$$\begin{aligned} \frac{1}{T} \int_0^T I_0^2 dt &= I_0^2 & \cos 2\omega t &= \cos^2 \omega t - \sin^2 \omega t \\ & & &= 1 - 2\sin^2 \omega t \\ \frac{1}{T} \int_0^T I_{mn}^2 \sin^2(n\omega t + \theta_n) dt & & \Rightarrow \sin^2 \omega t &= \frac{1 - \cos 2\omega t}{2} \\ &= \frac{I_{mn}^2}{2T} \int_0^T (1 - \cos 2(n\omega t + \theta_n)) dt = \left(\frac{I_{mn}}{\sqrt{2}}\right)^2 = |I_n|^2 \\ \frac{1}{T} \int_0^T I_0 I_{mn} \sin(n\omega t + \theta_n) dt &= 0 & \text{sin の積分は 0} \\ \frac{1}{T} \int_0^T I_{mh} I_{mk} \sin(h\omega t + \theta_h) \sin(k\omega t + \theta_k) dt &= 0 \quad (h \neq k) \end{aligned}$$

周波数が異なる sin 同士の積分は 0

3

$$\begin{aligned} \frac{1}{T} \int_0^T I_{mh} I_{mk} \sin(h\omega t + \theta_h) \sin(k\omega t + \theta_k) dt \\ &= -\frac{I_{mh} I_{mk}}{2T} \int_0^T \cos((h+k)\omega t + (\theta_h + \theta_k)) - \cos((h-k)\omega t + (\theta_h - \theta_k)) dt \\ &= -\frac{I_{mh} I_{mk}}{2T} \left\{ \frac{1}{h+k} [\sin((h+k)\omega T + (\theta_h + \theta_k))]_0^T - \frac{1}{h-k} [\sin((h-k)\omega T + (\theta_h - \theta_k))]_0^T \right\} \\ &= -\frac{I_{mh} I_{mk}}{2T} \left[ \frac{1}{h+k} \{ \sin((h+k)\omega T + (\theta_h + \theta_k)) - \sin(\theta_h + \theta_k) \} \right. \\ &\quad \left. - \frac{1}{h-k} \{ \sin((h-k)\omega T + (\theta_h - \theta_k)) - \sin(\theta_h - \theta_k) \} \right] \\ &= -\frac{I_{mh} I_{mk}}{2T} \left[ \frac{1}{h+k} \{ \sin((h+k)2\pi + (\theta_h + \theta_k)) - \sin(\theta_h + \theta_k) \} \right. \\ &\quad \left. - \frac{1}{h-k} \{ \sin((h-k)2\pi + (\theta_h - \theta_k)) - \sin(\theta_h - \theta_k) \} \right] \\ &= 0 \end{aligned}$$

4

$$\begin{aligned} \frac{1}{T} \int_0^T i(t)^2 dt &= \frac{1}{T} \int_0^T (I_0^2 + I_{m1}^2 \sin^2(\omega t + \theta_1) + I_{m2}^2 \sin^2(2\omega t + \theta_2) + \dots) \\ &\quad + 2I_0 (I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &\quad + 2I_{m1} \sin(\omega t + \theta_1) (I_{m2} \sin(2\omega t + \theta_2) + \dots) \\ &= I_0^2 + \frac{I_{m1}^2}{2} + \frac{I_{m2}^2}{2} + \dots \\ &= I_0^2 + \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \dots \\ &= I_1^2 + I_2^2 + \dots \end{aligned}$$

よって

$$|I| = \sqrt{I_0^2 + |I_1|^2 + |I_2|^2 + \dots}$$

5

[ 例題 31.2 ]

$$\begin{aligned} v(t) &= 4\sqrt{2} \sin(t - 30^\circ) + 2\sqrt{2} \sin(2t + 30^\circ) \\ &= \sqrt{2}|V_1| \quad = \sqrt{2}|V_2| \\ &\quad + 2\sqrt{2} \sin(3t - 30^\circ) + \sqrt{2} \sin(4t - 60^\circ) \\ &= \sqrt{2}|V_3| \quad = \sqrt{2}|V_4| \end{aligned}$$

$$|V_1| = 4$$

$$|V_2| = 2$$

$$|V_3| = 2$$

$$|V_4| = 1$$

よって

$$\begin{aligned} |V| &= \sqrt{|V_1|^2 + |V_2|^2 + |V_3|^2 + |V_4|^2} \\ &= \sqrt{4^2 + 2^2 + 2^2 + 1^2} = 5 \end{aligned}$$

6

**24 非正弦波交流**  
**24.5 波形率・波高率・ひずみ率**

$i(t) = I_0 + I_{m1} \sin(\omega t - \theta_1) + I_{m2} \sin(2\omega t - \theta_2) + \dots$

$I_1 = \frac{I_{m1}}{\sqrt{2}}, I_2 = \frac{I_{m2}}{\sqrt{2}}$

**波形率** =  $\frac{\text{実効値}}{\text{平均値}} = \frac{|I|}{I_a} = \frac{\sqrt{\frac{1}{T} \int_0^T i^2 dt}}{\frac{1}{T} \int_0^T |i| dt} = \frac{\sqrt{I_0^2 + I_1^2 + \dots}}{\frac{1}{T} \int_0^T |i| dt}$

**波高率** =  $\frac{\text{最大値}}{\text{実効値}} = \frac{I_m}{|I|} = \frac{I_m}{\sqrt{\frac{1}{T} \int_0^T i^2 dt}} = \frac{I_m}{\sqrt{I_0^2 + I_1^2 + \dots}}$

**ひずみ率** =  $\frac{\text{全高調波の実効値}}{\text{基本波の実効値}} = \frac{\sqrt{|I_2|^2 + |I_3|^2 + \dots}}{|I_1|}$

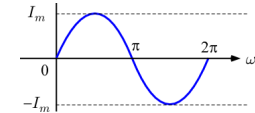
7

**正弦波**  $i(t) = I_m \sin \omega t$

**波形率** =  $\frac{\frac{I_m}{\sqrt{2}}}{\frac{2}{\pi} I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$

**波高率** =  $\frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.41$

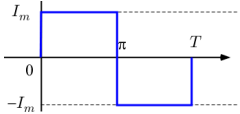
**ひずみ率** =  $\frac{0}{\frac{I_m}{\sqrt{2}}} = 0$      $2\omega t, 3\omega t, \dots$  はない



8

**方形波**

$i(t) = \begin{cases} I_m & (0 < t < \frac{T}{2}) \\ -I_m & (\frac{T}{2} < t < T) \end{cases}$



**波形率** =  $\frac{I_m}{I_m} = 1$

**波高率** =  $\frac{I_m}{I_m} = 1$

$i(t) = \frac{8}{T} I_m \left( \sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \dots \right)$

**ひずみ率** =  $\frac{\sqrt{\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \dots}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \dots}}{1} = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \dots}$

9

[例題 31.2]

$v(t) = \frac{4\sqrt{2}}{2} \sin(t - 30^\circ) + \frac{2\sqrt{2}}{2} \sin(2t + 30^\circ)$   
 $= \sqrt{2}|V_1| \quad = \sqrt{2}|V_2|$   
 $+ \frac{2\sqrt{2}}{2} \sin(3t - 30^\circ) + \frac{\sqrt{2}}{2} \sin(4t - 60^\circ)$   
 $= \sqrt{2}|V_3| \quad = \sqrt{2}|V_4|$

**ひずみ率**

$k = \frac{\sqrt{|V_2|^2 + |V_3|^2 + |V_4|^2}}{|V_1|}$   
 $= \frac{\sqrt{2^2 + 2^2 + 1^2}}{4} = \frac{3}{4}$

10

**第 24 章 : 非正弦波交流**

24.4 ひずみ波の実効値  
 24.5 ひずみ率

キーワード : ひずみ波の実効値, ひずみ率

学習目標 : ひずみ波の実効値, ひずみ率を計算できる。  
 ひずみ波の交流電力を求めることができる。

11