第 24 章：非正弦波交流

24.6 ひずみ波交流電力

キーワード：ひずみ波交流電力

学習目標：ひずみ波の交流電力を求めることができる。

`1`

---

24 非正弦波交流
24.6 ひずみ波交流電力

【復習】（単相）交流電力

$$e(t) = E_m \sin(\omega t + \theta_n)$$
$$i(t) = I_m \sin(\omega t + \theta_n - \varphi)$$

$$|E| = \frac{E_m}{\sqrt{2}}$$
$$|I| = \frac{I_m}{\sqrt{2}}$$

瞬時電力 $p = i \cdot e$ [W]

有効電力 $P = \dfrac{1}{T} \displaystyle\int_0^T p\, dt = |I||E| \cos\varphi$ [W]

無効電力 $P_r = |I||E| \sin\varphi$ [Var]

皮相電力 $P_a = |I||E|$ [VA]

力率 $\cos\varphi = \dfrac{P}{P_a}$

`2`

ひずみ波交流電力

$$e = E_0 + \sum_{n=1}^{\infty} E_{mn} \sin(n\omega t + \theta_n)$$
$$i = I_0 + \sum_{n=1}^{\infty} I_{mn} \sin(n\omega t + \theta_n - \varphi_n)$$

瞬時電力

$$p = i \cdot e$$
$$= E_0 I_0 + \sum_{n=1}^{\infty} E_{mn} I_{mn} \sin(n\omega t + \theta_n) \sin(n\omega t + \theta_n - \varphi_n)$$
$$+ \sum_{n=1}^{\infty} \{ E_0 I_{mn} \sin(n\omega t + \theta_n - \varphi_n) + I_0 E_{mn} \sin(n\omega t + \theta_n) \}$$
$$+ \sum_{\substack{n=1 \\ n \neq k}}^{\infty} \sum_{k=1}^{\infty} E_{mn} I_{mk} \sin(n\omega t + \theta_n) \sin(k\omega t + \theta_k - \varphi_k)$$

`3`

有効電力

$$P = \frac{1}{T} \int_0^T p\, dt$$
$$= \frac{1}{T} \int_0^T E_0 I_0 dt + \underbrace{\frac{1}{T} \int_0^T \sum_{n=1}^{\infty} E_{mn} I_{mn} \sin(n\omega t + \theta_n) \sin(n\omega t + \theta_n - \varphi_n) dt}_{= \sum_{n=1}^{\infty} \frac{E_{mn} I_{mn} \cos\varphi_n}{2}}$$
$$\quad = E_0 I_0$$
$$+ \underbrace{\sum_{n=1}^{\infty} \{ E_0 I_{mn} \sin(n\omega t + \theta_n - \varphi_n) + I_0 E_{mn} \sin(n\omega t + \theta_n) \}}_{= 0}$$
$$+ \underbrace{\sum_{\substack{n=1 \\ n \neq k}}^{\infty} \sum_{k=1}^{\infty} E_{mn} I_{mk} \sin(n\omega t + \theta_n) \sin(k\omega t + \theta_k - \varphi_k)}_{= 0}$$
$$= E_0 I_0 + \sum_{n=1}^{\infty} \frac{E_{mn}}{\sqrt{2}} \frac{I_{mn}}{\sqrt{2}} \cos\varphi_n$$

`4`

$\dfrac{1}{T} \displaystyle\int_0^T \sum_{n=1}^{\infty} E_{mn} I_{mn} \sin(n\omega t + \theta_n) \sin(n\omega t + \theta_n - \varphi_n) dt = \sum_{n=1}^{\infty} \dfrac{E_{mn} I_{mn} \cos\varphi_n}{2}$ の証明

$\sin(\alpha)\sin(\beta) = -\dfrac{1}{2}\{\cos(\alpha+\beta) - \cos(\alpha-\beta)\}$ を用いて

$$-\frac{E_{mn} I_{mn}}{2T} \sum_{n=1}^{\infty} \int_0^T (\cos(2n\omega t + 2\theta_n - \varphi_n) - \cos(\varphi_n))\, dt$$
$$= -\frac{E_{mn} I_{mn}}{2T} \sum_{n=1}^{\infty} \left[\frac{1}{2n\omega} \sin(2n\omega t + 2\theta_n - \varphi_n) - \cos(\varphi_n)t \right]_0^T$$
$$= -\frac{E_{mn} I_{mn}}{2T} \sum_{n=1}^{\infty} \left(\underbrace{\frac{1}{2n\omega} \sin(2n\omega T + 2\theta_n - \varphi_n) - \cos(\varphi_n)T - \frac{1}{2n\omega} \sin(2\theta_n - \varphi_n)}_{= \frac{1}{2n\omega} \sin(2\theta_n - \varphi_n)} \right)$$
$$= \sum_{n=1}^{\infty} \frac{E_{mn} I_{mn}}{2} \cos(\varphi_n)$$

`5`

有効電力

$$P = E_0 I_0 + \sum_{n=1}^{\infty} |E_n||I_n| \cos\varphi_n$$
$$= E_0 I_0 + |E_1||I_1| \cos\varphi_1 + |E_2||I_2| \cos\varphi_2 + \cdots$$

$$|E_n| = \frac{E_{mn}}{\sqrt{2}}$$
$$|I_n| = \frac{I_{mn}}{\sqrt{2}}$$

有効電力（平均電力）は周波数が等しい電圧と電流との間の有効電力の総和

皮相電力

$$P_a = |E||I|$$
$$= \sqrt{(E_0^2 + |E_1|^2 + |E_2|^2 + \cdots) \cdot (I_0^2 + |I_1|^2 + |I_2|^2 + \cdots)}$$

`6`

力率

$$\cos\varphi = \frac{\text{有効電力}}{\text{皮相電力}} = \frac{P}{P_a}$$

$$= \frac{E_0 I_0 + |E_1||I_1|\cos\varphi_1 + |E_2||I_2|\cos\varphi_2 + \cdots}{\sqrt{(E_0^2 + |E_1|^2 + |E_2|^2 + \cdots)\cdot(I_0^2 + |I_1|^2 + |I_2|^2 + \cdots)}}$$

7

[例題31.2]

$$v(t) = 4\sqrt{2}\sin(t - 30°) + 2\sqrt{2}\sin(2t + 30°) + 2\sqrt{2}\sin(3t - 30°) + \sqrt{2}\sin(4t - 60°)$$
$$i(t) = 5\sqrt{2}\sin(t - 30°) + 3\sqrt{2}\sin(2t + 30°) + \sqrt{2}\sin(3t + 30°) + \sqrt{2}\sin(4t + 30°)$$

実効値

$$V = \sqrt{\left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{16 + 4 + 4 + 1}$$

$$= \sqrt{25} = 5 \text{ [V]}$$

実効値

$$I = \sqrt{\left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{3\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{25 + 9 + 1 + 1}$$

$$= \sqrt{36} = 6 \text{ [A]}$$

8

$$v(t) = 4\sqrt{2}\sin(t - 30°) + 2\sqrt{2}\sin(2t + 30°) + 2\sqrt{2}\sin(3t - 30°) + \sqrt{2}\sin(4t - 60°)$$
$$i(t) = 5\sqrt{2}\sin(t - 30°) + 3\sqrt{2}\sin(2t + 30°) + \sqrt{2}\sin(3t + 30°) + \sqrt{2}\sin(4t + 30°)$$

有効電力

$$P = 4 \times 5\cos(-30° - (-30°)) + 2 \times 3\cos(30° - (30°))$$
$$\qquad + 2 \times 1\cos(-30° - (30°)) + 1 \times 1\cos(-60° - (30°))$$

$$= 4 \times 5\cos(0°) + 2 \times 3\cos(0°) + 2 \times 1\cos(-60°) + 1 \times 1\cos(-90°)$$

$$= 4 \times 5\cos(0°) + 2 \times 3\cos(0°) + 2 \times 1\cos(60°) + 1 \times 1\cos(90°)$$

$$= 4 \times 5 + 2 \times 3 + 2 \times 1 \times \frac{1}{2} + 1 \times 1 \times 0$$

$$= 20 + 6 + 1 \qquad \boxed{\cos(-\theta) = \cos(\theta)}$$

$$= 27 \text{ [W]}$$

力率

$$\cos\varphi = \frac{P}{V\cdot I} = \frac{27}{5\times 6} = 0.9$$

9

第24章：非正弦波交流

24.6 ひずみ波交流電力

キーワード：**ひずみ波交流電力**

学習目標：**ひずみ波の交流電力を求めることができる。**

10

2